CIS260-201/204–Spring 2008
## Recitation 10 Supplementary Exercises
Friday, April 4

1. In this problem, we examine the *complete bipartite graphs*. Recall that a graph G is *bipartite* if $V(G)$ can be partitioned so that there are no edges in each partition. If for every pair of vertices from each partition, there is an edge between them, then G is *complete bipartite*, and we denote G by $K_{m,n}$, where m is the number of vertices in one partition and n is the number of vertices in the other partition.

   (a) How many edges are there in $K_{m,n}$? Explain.

   (b) How many graphs are the spanning subgraphs of $K_{m,n}$? Explain.

   (c) How many graphs are the induced subgraphs of $K_{m,n}$? Explain.

   (d) Find and prove $\omega(K_{m,n})$, where $\omega(G)$ is the clique number of G.

   (e) Find and prove $\alpha(K_{m,n})$, where $\alpha(G)$ is the independence number of G.

(f) What special property does each vertex partition in $K_{m,n}$ have. Explain.

(g) Recall the *Hamiltonian path/cycle* problem. A path P in G that contains all the vertices of G is called a *Hamiltonian path*. A cycle of G that contains all the vertices in G is called a *Hamiltonian cycle*.

    i. Prove that $K_{m,n}$ has a Hamiltonian path if and only if $|m - n| \leq 1$.

    ii. Prove that $K_{m,n}$ has a Hamiltonian cycle if and only if $m = n$.

    iii. Argue that if $K_{m,n}$ has a Hamiltonian cycle, then $K_{m,n}$ has a Hamiltonian path.

2. Recall the Petersen's graph:
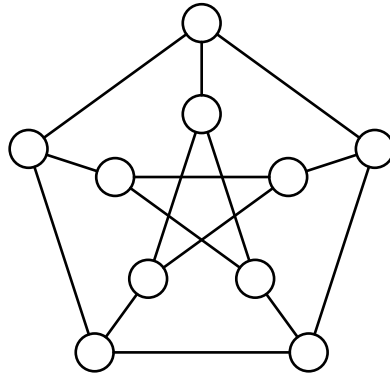
Prove that this graph is nonplanar by finding either a subdivision of K_5 or a subdivision of $K_{3,3}$ as a subgraph. [**Hint**: It is a good idea to label vertices so you can keep track of which vertices are in the subgraph.]

3.   (a) Let G be a k-regular graph with n vertices. Find k in terms of n so that G is planar.

    (b) For which n are there no k-regular graph with n vertices that is planar, where k is any positive integer?

4. (a) Give an example of a sample space in which all of the elements have probability 1.

   (b) Give an example of a sample space with four elements in which one of the elements has probability equal to 1.

5. Let (S, P) be the sample space in which $S = \{1, 2, 3, 4, 5, 6\}$ and $\Pr(i) = i \Pr(1)$. For example, $\Pr(2) = 2 \Pr(1)$. Find $\Pr(s)$ for each $s \in S$.

6. A *palindrome* is a sequence of letters where the expression is unchanged upon reversing order. For example, *racecar* is a palindrome. Consider n-bit binary strings.

   (a) How many n-bit binary strings are palindromes?

   (b) An n-bit binary string is chosen at random. What is the probability that it is a palindrome?

7. On R 04/03 we started discussing about Probability Theory. As you might notice, we will need to go back and use the counting principles we learned in Section 7 of the book. Some of the counting problems also require the binomial coefficients in Section 16. In preparation for this, this problem gives you some more practice on counting correctly. We will deal with a deck of cards. If you happen to encounter many kinds of cards, the one we will use here and in class is the standard 52-card deck with four suits and thirteen faces in each suit. Suppose further that we are playing poker, in which we need to draw five cards from the deck. Recall from class that we can draw five cards in $\binom{52}{5}$ ways. We call the five cards a hand.

In each problem below, please explain the steps you proceed toward your answer.

(a) In how many ways can you draw a hand having four of a kind, i.e., four cards of the same face and one other card?

(b) In how many ways can you draw a hand having three of a kind (and exactly three)?

(c) In how many ways can you draw a hand having a pair (two of a kind)?

(d) In how many ways can you draw a hand of two pairs?

(e) In how many ways can you draw a flush, i.e., five cards of the same suit?

(f) In how many ways can you draw a full house, i.e., three of a kind and a pair?

(g) In how many ways can you draw a straight flush, i.e., i.e., five cards of the same suit with face values in order? Note that Ace can be combined with either $2, 3, 4, 5$ or $10, J, Q, K$.?

(h) [Extra Credit!] In how many ways can you draw a straight that is not a straight flush, i.e., a straight in which all the cards are not of the same suit?

(i) [Extra Credit!!!] In how many ways can you draw a regular hand, i.e., anything other than above, also known as "high card"? [**Caution**: Don't do this if you can't do 7(h)!]