Name: ______________________________ Section: 201 204

CIS260-201/204–Spring 2008

Recitation 9 Supplementary Exercises

Friday, March 28

1. Recall the *is-connected-to* relation on the vertices of a graph. u is connected to v if there is a (u, v)-path, i.e., a path from u to v. Show that is-connected-to needs not be irreflexive or antisymmetric.

2. Recall the definition of *connected*:
   Definition: A graph is *connected* if for all $u, v \in V(G)$, u is connected to v.
   Prove that K_n, the complete graph on n vertices, is connected.

3. Prove or disprove: A graph is connected if and only if there is a path that contains every pair of vertices in G.

4. If you know Java, you probably know the term "overloading," which refers to the use of an identical name for different methods. Similar story happens in the graph terminology. Recall the definition of *path*:
Definition: A *path* in a graph is a walk in which no vertex is repeated.
The term *path* can refer to another situation. We also call a graph on n vertices whose structure is simply a path a *path graph*, denoted P_n. Formally,
Definition: A *path* is a graph with vertex set $V = \{v_1, v_2, \ldots, v_n\}$ and edge set $E = \{v_i v_{i+1} \mid 1 \leq i < n\}$. A path on n vertices is denoted P_n.

   (a) Prove that for every n, P_n is a tree. [**Hint**: What's the definition of a tree?]

   (b) Prove that for every $n \geq 2$, P_n has exactly two leaves

      i. directly.

      ii. by induction.

5. Here's another overload. Recall the definition of *cycle*:
Definition: A *cycle* is a walk of length at least three in which the first and last vertex are the same, but no other vertices are repeated.
The term *cycle* can refer to another situation. We also call a graph on n vertices whose structure is simply a cycle a *cycle graph*, denoted C_n.
In recitation we know that every cycle (graph) is 2-regular, meaning that every vertex has degree 2. Is the converse true, i.e., if G is a graph in which every vertex has degree 2, is G necessarily a cycle? If true, prove it. If false, you know the deal—give a counterexample.

6. Let T be a tree. Prove that the average degree of a vertex in T is less than 2. [**Hint**: What kind of proofs that you were tempted to do on trees but couldn't, but you can use it now?]

7. Complete the proof a theorem done in class on T 03/25: Let G be an acyclic graph on n vertices. If G has $n-1$ edges, then G is connected. Explain how you reach each step in your proof. [**Hint**: You may use the fact we proved in recitation today: If G is a forest with n vertices and c components, then there are $n-c$ edges in G.]

8. Recall the seven bridges of Königsberg. Is it possible to walk the seven bridges so that you cross every bridge exactly twice, once in each direction? If so, give a valid walk below (by drawing). If not, argue why.

9. Let G be a $(2k+1)$-regular graph, where k is a natural number. Prove that G must have an even number of vertices.