Cornell University
Department of Computer Science

Reconciling Exhaustive Pattern Matching with Objects

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PLDI 2013 | Wed, June 19, 2013 | Seattle, WA

This paper

integrating

pattern matching

(makes code concise & safer)

with

object-oriented programming

(helps software scale)

Pattern matching in OCaml: concise & safer code

```
type list = Nil | Cons of int * list

match l with
| Nil -> ...
| Cons(x1, Cons(x2, l')) -> ...
| Cons(x1, Cons(x2, Cons(x3, l'))) -> ...
```

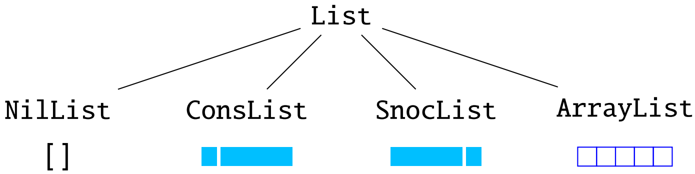
$\texttt{list} = \texttt{Nil} \uplus \texttt{Cons}$

$$\begin{aligned} \texttt{Cons} &: \texttt{int} \times \texttt{list} \to \texttt{list} \\ \texttt{Cons}^{-1} &: \texttt{list} \to \texttt{int} \times \texttt{list} \end{aligned}$$

Exhaustiveness: `Cons(17, Nil)` not matched → warning
Nonredundancy: Third arm unnecessary → warning

Object-oriented programming: flexible & scalable

data abstraction



multiple implementations

```
switch (l) {      // Want this!
case Nil(): ...
case Cons(int x1, Cons(int x2, List tl)): ...
case Cons(int x1, Cons(int x2, Cons(int x3, List tl))): ...
}
```

Have your cake and eat it too?

Problem statement

Can we satisfy all these goals **without** violating data abstraction?

1. implementation-oblivious pattern matching
2. verification of exhaustive and nonredundant pattern matching

Comparison: prior & our approaches

approach		data abstraction	constructors usable as patterns	multiple impls of data types	exhaustiveness check
ML pattern matching			✓		✓
views	[W 87]	✓			
active patterns in F#	[SNM 07]	✓			✓
extractors	[EOW 07]	✓	(✓)	✓	
sealed classes in Scala	[OSV 08]		✓		✓
JMatch 1.1.6	[LM 03]	✓	✓	✓	
JMatch 2.0		✓	✓	✓	✓

Modal abstraction in JMatch 1.1.6

`list` `Cons` in ~Java:

$\texttt{Cons} : \texttt{int} \times \texttt{list} \rightarrow \texttt{list}$

```
Cons(int x, List l) {
  this.hd = x;
  this.tl = l;
}
```

$\texttt{Cons}^{-1} : \texttt{list} \rightarrow \texttt{int} \times \texttt{list}$

```
(int * List) cons() {
  return (this.hd, this.tl);
}
```

Different views of the same relation:

$$\{(this, x, l) \in \texttt{Cons} \times \texttt{int} \times \texttt{List} \mid this.\texttt{hd} = x \wedge this.\texttt{tl} = l\}$$

Modal abstraction in JMatch 1.1.6

`list` Cons in ~Java:

Cons : int × list → list

```
Cons(int x, List l) {
  this.hd = x;
  this.tl = l;
}
```

$\texttt{Cons}^{-1}$: list → int × list

```
(int * List) cons() {
  return (this.hd, this.tl);
}
```

Different views of the same relation:

$$\{(this, x, l) \in \texttt{Cons} \times \texttt{int} \times \texttt{List} \mid this.\texttt{hd} = x \wedge this.\texttt{tl} = l\}$$

JMatch 1.1.6:

```
Cons(int x, List l) returns(x, l) (
  this.hd = x && this.tl = l
)
```

Modal abstraction in JMatch 1.1.6

list Cons in ~Java:

Cons : int × list → list

```
Cons(int x, List l) {
  this.hd = x;
  this.tl = l;
}
```

$\mathtt{Cons}^{-1}$: list → int × list

```
(int * List) cons() {
  return (this.hd, this.tl);
}
```

Different views of the same relation:

$$\{(this, x, l) \in \mathtt{Cons} \times \mathtt{int} \times \mathtt{List} \mid this.\mathtt{hd} = x \land this.\mathtt{tl} = l\}$$

JMatch 1.1.6:

```
Cons(int x, List l) returns(x, l) (
  this.hd = x && this.tl = l
)
```

Modal abstraction in action

```
Cons(int x, List l) returns(x, l) (
  this.hd = x && this.tl = l
)
```

```
// forward mode
let List l = Cons(hd, tl);
// backward mode
let l = Cons(int hd, List tl);
```

```
List l0 = Nil();                  // l0 = []
List l1 = Cons(17, l0);           // l1 = [17; []]
List l2 = Cons(42, l1);           // l2 = [42; [17; []]]

switch (l2) {
case Nil(): ...
case Cons(int x1, List l): ... // x1 ↦ 42, l ↦ [17; []]
case Cons(int x1, Cons(int x2, List l)): ...
}                          // x1 ↦ 42, x2 ↦ 17, l ↦ Nil()
```

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Implementation-oblivious pattern matching

```
// JMatch 1.1.6
Cons(int x, List l) returns(x, l) (
  this.hd = x && this.tl = l
)
```

Problem: `Cons` constructors belong to the `Cons` class.

Solution: Declare `Cons` independently of implementations.

JMatch 2.0 — Constructors in interfaces

Constructors can be declared in interfaces:

```
interface List {
  constructor nil() returns();
  constructor cons(int x, List l) returns(x, l);
}
```

JMatch 2.0 — Constructors in interfaces

Constructors can be declared in interfaces:

```
interface List {
  constructor nil() returns();
  constructor cons(int x, List l) returns(x, l);
}
```

```
class Nil implements List {
  public constructor nil() returns() ( true )
  public constructor cons(int x, List l)
    returns(x, l) ( false )
}
class Cons implements List {
  int hd; List tl;
  public constructor nil() returns() ( false )
  public constructor cons(int x, List l)
    returns(x, l) ( this.hd = x && this.tl = l )
}
```

Another `List` implementation

snoc list:


```
class Snoc implements List {
  List hd;
  int tl;

  public constructor nil() returns() ( false )
  public constructor cons(int x, List l) returns(x, l) (
       l = nil() && this.hd = l && this.tl = x
    |  l = Snoc(List lhd, int ltl)
         && this.hd = cons(x, lhd) && this.tl = ltl
  )
  Snoc(List l, int x) returns(l, x) (
    this.hd = l && this.tl = x
  )
}
```

Another `List` implementation

snoc list:


```
class Snoc implements List {
  List hd;
  int tl;

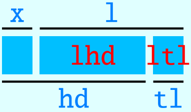
  public constructor nil() returns() ( false )
  public constructor cons(int x, List l) returns(x, l) (
      l = nil() && this.hd = l && this.tl = x
    |  l = Snoc(List lhd, int ltl)
        && this.hd = cons(x, lhd) && this.tl = ltl
  )
  Snoc(List l, int x) returns(l, x) (
    this.hd = l && this.tl = x
  )
}
```

Another `List` implementation

snoc list: 


```
class Snoc implements List {
  List hd;
  int tl;

  public constructor nil() returns() ( false )
  public constructor cons(int x, List l) returns(x, l) (
        l = nil() && this.hd = l && this.tl = x
     | l = Snoc(List lhd, int ltl)
          && this.hd = cons(x, lhd) && this.tl = ltl
  )
  Snoc(List l, int x) returns(l, x) (
    this.hd = l && this.tl = x
  )
}
```



Another `List` implementation

snoc list:


```
class Snoc implements List {
  List hd;
  int tl;

  public constructor nil() returns() ( false )
  public constructor cons(int x, List l) returns(x, l) (
      l = nil() && this.hd = l && this.tl = x
    | l = Snoc(List lhd, int ltl)
       && this.hd = cons(x, lhd) && this.tl = ltl
  )
  Snoc(List l, int x) returns(l, x) (
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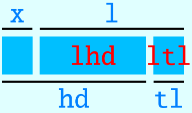
  )
}
```



JMatch 2.0 — Equality constructors

```
l = Snoc(List lhd, int ltl)
l.equals(Snoc(List lhd, int ltl))  // equals multimodal
```

Problem: `l` is nonempty but might not be a `Snoc`.

Solution:

- Convert `l` into a `Snoc` first (always succeeds).
- Do this implicitly; don't bother programmer.

Equality constructors specify *how* the conversion should be done.

```
public constructor equals(List l) (
  l = cons(int lhd, List ltl) && cons(lhd, ltl)
)
```

Have your cake and eat it too?

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Can we satisfy all these goals **without** violating data abstraction?

1. implementation-oblivious pattern matching ✓
2. verification of exhaustive and nonredundant pattern matching

Checking exhaustiveness and nonredundancy

```
interface List {
  constructor nil() returns();
  constructor cons(int x, List l) returns(x, l);
}
```

```
switch (l) {
case nil(): ...
case cons(int hd, List tl): ...
}
```

1. `switch` exhaustive?
2. Any `case` redundant?

Invariants

1. `nil` and `cons` can construct every `List`.
2. No value can be constructed by both `nil` and `cons`.

$$\texttt{List} = \texttt{nil} \uplus \texttt{cons}$$

Invariants

1. `nil` and `cons` can construct every `List`.
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$$\texttt{List} = \texttt{nil} \uplus \texttt{cons}$$

`|` represents disjoint disjunction:

```
invariant(this = nil() | this = cons(_, _));
```

Add invariants to interfaces.

Invariants

1. `nil` and `cons` can construct every `List`.
2. No value can be constructed by both `nil` and `cons`.

$$\texttt{List} = \texttt{nil} \uplus \texttt{cons}$$

| represents disjoint disjunction:

```
invariant(this = nil() | this = cons(_, _));
```

| for disjoint patterns:

```
invariant(this = nil() | cons(_, _));
```

Add invariants to interfaces.

Invariants not enough

```
interface List {
  constructor nil() returns();
  constructor cons(int x, List l) returns(x, l);
  constructor snoc(List l, int x) returns(l, x);
}
```

```
switch (l) {
case nil(): ...
case snoc(List hd, int tl): ...
}
```

1. `switch` exhaustive?
2. Any `case` redundant?

Matching precondition

Know: exhaustiveness of

```
switch (l) {
case nil(): ...
case cons(int hd, List tl): ...
}
```

Want: exhaustiveness of

```
switch (l) {
case nil(): ...
case snoc(List hd, int tl): ...
}
```

If `cons` matches , `snoc` matches.

Matching precondition

Know: exhaustiveness of

```
switch (l) {
case nil(): ...
case cons(int hd, List tl): ...
}
```

Want: exhaustiveness of

```
switch (l) {
case nil(): ...
case snoc(List hd, int tl): ...
}
```

If `cons` matches, `snoc` matches.

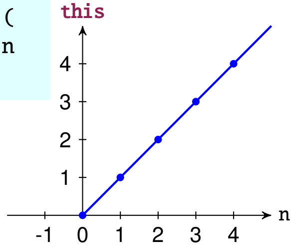
↑

matching precondition `this = cons(_, _)`

Partial functions

Natural numbers represented by integers:

```
ZNat(int n) returns(n) (
  n >= 0 && this.rep = n
)
```



the ZNat relation

Partial functions

Natural numbers represented by integers:

```
ZNat(int n) returns(n) (
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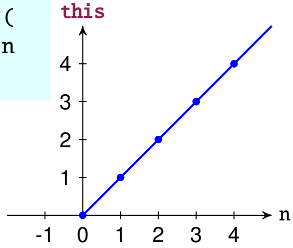
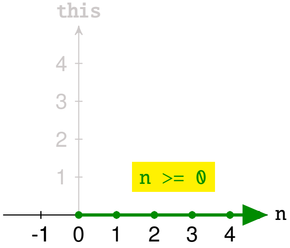
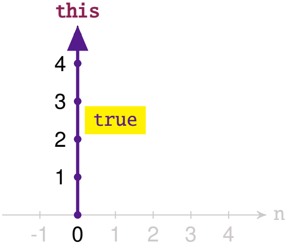


the ZNat relation



matching precondition for **returns**(**this**)



matching precondition for **returns**(n)

Matches clauses

In `ZNat`, matching precondition for

- forward mode: `n >= 0`
- backward mode: `true`

Writing a matching precondition per mode is tedious.

Matches clauses

In `ZNat`, matching precondition for

- forward mode: `n >= 0`
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Writing a matching precondition per mode is tedious.

Modal abstraction → **consolidated** method body
??? → **consolidated** matching precondition

Matches clauses

In `ZNat`, matching precondition for

- forward mode: `n >= 0`
- backward mode: `true`

Writing a matching precondition per mode is tedious.

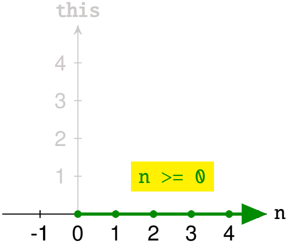
Modal abstraction → **consolidated** method body
Matches clause → **consolidated** matching precondition

```
ZNat(int n) matches(n >= 0) returns(n) (
  n >= 0 && this.rep = n
)
```

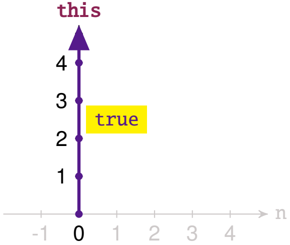
How to recover individual matching preconditions?

Specifying & interpreting a matches clause

matching precondition for `returns(this)`

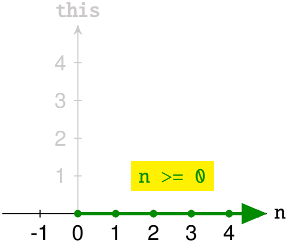


matching precondition for `returns(n)`

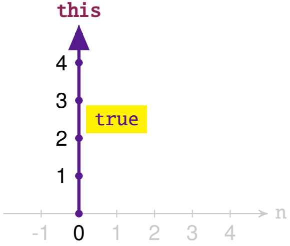


Specifying & interpreting a matches clause

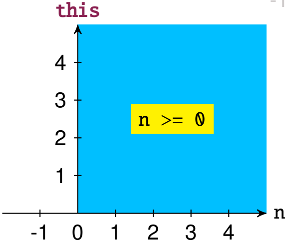
matching precondition for **returns**(**this**)



matching precondition for **returns**(n)



Use projections to recover individual matching preconditions.



the matches clause

Verification summary

matching precondition	$\Rightarrow$	**method body**
forward mode of `ZNat`: $n \geq 0$	$\Rightarrow$	$\exists rep : n \geq 0 \land rep = n$
backward mode of `ZNat`: true	$\Rightarrow$	$\exists n : n \geq 0 \land rep = n$ (need invariant $rep \geq 0$)

```
class ZNat implements Nat {
  private invariant(rep >= 0);
  ZNat(int n) matches(n >= 0) returns(n) (
    n >= 0 && this.rep = n
  )
  ...
}
```

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Implementation

Pattern matching features:

- Translate to Java (extends JMatch 1.1.6).
- Original semantics redefined to handle implicit equality constructor calls.

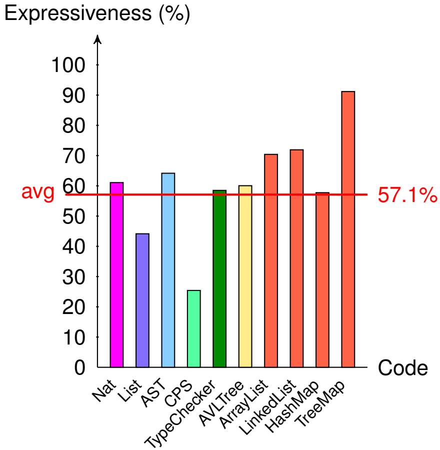
Verification:

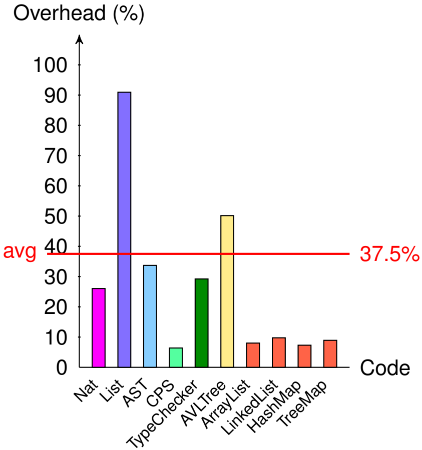
- Encode verification conditions for Z3 theorem prover.

Evaluation

On code examples (include Java collections framework):

- Implemented concisely in JMatch & Java, compare token counts.
- Verification correctness and overhead during compilation.





```
interface Tree {
  invariant(this = leaf() | branch(_,_,_));
  constructor leaf() matches(height() = 0) ensures(height() = 0);
  constructor branch(Tree l, int v, Tree r) matches(height() > 0)
    ensures(height() > 0 &&
      (height() = l.height() + 1 && height() > r.height() ||
       height() > l.height() && height() = r.height() + 1))
    returns(l, v, r);
  int height() ensures(result >= 0);
}
static Tree rebalance(Tree l, int v, Tree r) matches(true) ( // in AVLTree
  result = Branch(Branch(Tree a, int x, Tree b), int y,
                  Branch(Tree c, int z, Tree d))
    && ( l.height() - r.height() > 1 && d = r && z = v  // rot. from left
           && ( l = branch(Tree ll, y, c) && ll = branch(a, x, b) &&
                  ll.height() >= c.height()
              | l = branch(a, x, Tree lr) && lr = branch(b, y, c) &&
                  a.height() < lr.height())
       | r.height() - l.height() > 1 && a = l && x = v  // rot. from right
           && ( r = branch(Tree rl, z, d) && rl = branch(b, y, c) &&
                  rl.height() > d.height()
              | r = branch(b, y, Tree rr) && rr = branch(c, z, d) &&
                  b.height() <= rr.height()))
  | abs(l.height() - r.height()) <= 1 && result = Branch(l, v, r)
)
```

Takeaways

- Compact code for pattern matching possible in object-oriented settings
  - named and equality constructors, disjoint disjunctions
  - pattern disjunctions, tuples
- Verifying exhaustiveness and nonredundancy of pattern matching possible
  - invariants, multimodal matches clauses
  - ensures clauses, opaque matching preconditions

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http://www.cs.cornell.edu/projects/JMatch/

additional slides

Equality constructors in action

```
public constructor cons(int x, List l) returns(x, l) (
    l = nil() && this.hd = l && this.tl = x
  | l = Snoc(List lhd, int ltl) &&
      this.hd = cons(x, lhd) && this.tl = ltl
)
public constructor snoc(List l, int x) returns(l, x) (
  this.hd = l && this.tl = x
)
public constructor equals(List l) (
  l = cons(int lhd, List ltl) && cons(lhd, ltl)
)
```

```
List result = Snoc.cons(42, Cons.cons(17, Nil.nil()));
```

Convert `[17; []]` into a `Snoc`, calling `Snoc.cons(17, Nil.nil())`.
The conversion is `[[]; 17]`, so `lhd = []`, `ltl = 17`

Equality constructors in action

```
public constructor cons(int x, List l) returns(x, l) (
    l = nil() && this.hd = l && this.tl = x
  | l = Snoc(List lhd, int ltl) &&
      this.hd = cons(x, lhd) && this.tl = ltl
)
public constructor snoc(List l, int x) returns(l, x) (
  this.hd = l && this.tl = x
)
public constructor equals(List l) (
  l = cons(int lhd, List ltl) && cons(lhd, ltl)
)
```

```
List result = Snoc.cons(42, Cons.cons(17, Nil.nil()));
```

hd = cons(42, []) = [[]; 42], tl = 17

result = [[[]; 42]; 17]

Equality constructors in action

Generic equality constructors for any `List`:

```
public constructor equals(List l) (
    l = nil() && nil()
  | l = cons(int lhd, IntList ltl) && cons(lhd, ltl)
)
```

In action...

```
List l0 = Nil.nil();            // l0 = []
List l1 = Cons.cons(17, l0);    // l1 = [17; []]
List l2 = Snoc.cons(42, l1);    // l2 = [[[]; 42]; 17]
List l3 = Cons.cons(47, l2);    // l3 = [47; [42; [17; []]]]
```

Equality constructors in action

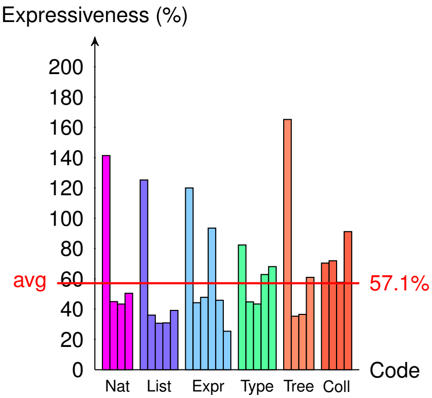
Generic equality constructors for any `List`:

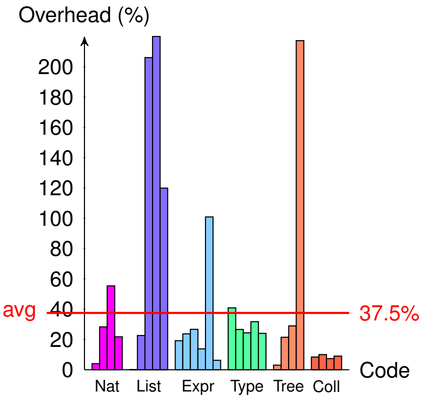
```
public constructor equals(List l) (
    l = nil() && nil()
  | l = cons(int lhd, IntList ltl) && cons(lhd, ltl)
)
```

In action...

```
List l0 = Nil.nil();             // l0 = []
List l1 = Cons.cons(17, l0);     // l1 = [17; []]
List l2 = Snoc.cons(42, l1);     // l2 = [[[]; 42]; 17]
List l3 = Cons.cons(47, l2);     // l3 = [47; [42; [17; []]]]
```

Evaluation


Expressiveness (%)
200
180
160
140
120
100
80
60
40
20
0
avg
57.1%
Code
Nat
List
Expr
Type
Tree
Coll


Overhead (%)
200
180
160
140
120
100
80
60
40
20
0
avg
37.5%
Code
Nat
List
Expr
Type
Tree
Coll